How does airflow speed (0, 2.5, 3.6, 7.1, 9.4, 17.6, 19.3, 21.4 ms^{-1}) affect the temperature drop ($^{\circ}C$) of hot water over a fixed time interval (300 s)?

Investigating the Impact of Airflow Speed on the Cooling and Evaporation Effect of Hot Water Under Forced Convection Conditions

Word Count: 2997

Contents

1 Exploration

1.1 Introduction

This investigation focuses on the topic "The particulate nature of matter." Convection is one of the primary modes of heat transfer. It involves the transfer of heat through a fluid (which can be a liquid or a gas) caused by the fluid's motion. This investigation will focus on heat transfer under forced convection conditions, where fluid motion is generated by an external source.

My fascination with this topic began during my adventures in DIY PC building. As I experimented with various fan configurations for my home computer, I became curious about the relationship between airflow speed and cooling efficiency. This interest wasn't limited to just computers. I started noticing forced convection in many aspects of daily life. From ceiling fans to industrial cooling systems, I realized how common and important this concept is.

1.2 Research Question

How does airflow speed (0, 2.5, 3.6, 7.1, 9.4, 17.6, 19.3, 21.4 ms^{-1}) affect the temperature drop ($^{\circ}C$) of hot water over a fixed time interval (300 s)?

1.3 Assumptions and Justifications

- **Assumption 1:** The room temperature remains constant throughout the experiment.

  Justification 1: This simplifies the analysis by ensuring that external temperature fluctuations do not influence the cooling rate, allowing for a clearer interpretation of the effect of airflow speed.

- **Assumption 2:** The airflow generated by the airflow generator is uniform around the water container.

  Justification 2: Uniform airflow ensures consistent forced convection conditions, making it easier to attribute changes in the cooling rate solely to variations in airflow speed.

- **Assumption 3:** The properties of water remain constant over the temperature range studied.

  Justification 3: The temperature range is relatively small, causing minimal variations in water properties.

- **Assumption 4:** The water is uniformly heated to 75 $^{\circ}C$ before the start of each trial.

  Justification 4: This provides a consistent starting point for each trial, reducing variability in the results.

- **Assumption 5:** Conduction is negligible compared to forced convection.

  Justification 5: The experimental setup is designed to emphasize forced convection. The insulation provided by the foam box reduces conduction.

1.4 Framework

The time-dependent temperature drop of hot water under varying airflow speeds is the focus of this investigation. This temperature decrease is influenced by two primary factors: convective heat transfer and evaporative cooling, both of which are functions of time and airflow speed.

A heat balance equation can be established to describe the heat exchange between the water and the environment. The total rate of heat loss at any given time t is the sum of the rates due to convective heat transfer and evaporative cooling (Çengel & Ghajar, 2015):

$$mc\frac{dT}{dt} = \dot{q}(t) = \dot{q}_{\text{conv}}(t) + \dot{q}_{\text{evap}}(t)$$

Where m is the mass of water (ml), c is the specific heat capacity of water ($J/kg \cdot K$), T is the temperature of the water at time t (K), $\dot{q}(t)$ is the total heat loss rate at time t (k), $\dot{q}_{\text{conv}}(t)$ is the convective heat transfer rate at time t (W), $\dot{q}_{\text{evap}}(t)$ is the evaporative cooling rate at time t (W).

The convective heat transfer rate at time t is given by (Çengel & Ghajar, 2015):

$$\dot{q}_{\text{conv}}(t) = h(v) \cdot A \cdot (T_s(t) - T_\infty)$$

Where $h(t) = h_0 \cdot v^n$ is the convective heat transfer coefficient (W/m^2K), which varies with airflow speed v and has an empirical exponent n. h_0 is a constant dependent on the specific conditions of the experiment, A is the surface area of the water m^2, and $T_s(t)$ is the surface temperature of the water at time t (K).

According to Newton's law of cooling (Davidzon, 2012), the surface temperature of the water $T_s(t)$ exponentially decays to the ambient temperature T_∞ over time. α is a constant related to the characteristics of the system.

$$T_s(t) = T_\infty + (T_i - T_\infty) \cdot e^{-\alpha t}$$

The evaporative cooling rate at time t is given by (Çengel & Ghajar, 2015):

$$\dot{q}_{\text{evap}}(t) = h_m(v) \cdot A \cdot (P_s(t) - P_\infty) \cdot L_v$$

Where $h_m(t) = h_{m0} \cdot v^m$ is the mass transfer coefficient (m/s) that also varies with airflow speed v and has an empirical exponent m, h_{m0} is a constant associated with the specific conditions of the experiment, and $P_s(t)$ is the vapor pressure at the water surface at time t (Pa).

The vapor pressure at the water surface at time t, $P_s(t)$, is related to the surface temperature $T_s(t)$. The Clausius-Clapeyron equation also describes this relationship (Feynman, Leighton, & Sands, 2015). It will be used for simplification:

$$P_s(t) = P_0 \cdot \exp\left(\frac{L_v}{R_v} \cdot (\frac{1}{T_0} - \frac{1}{T_s(t)})\right)$$

Where P_0 (Pa) is the saturated vapor pressure at the reference temperature T_0 (the boiling point of water at standard atmospheric pressure, $T_0 = 100\ ^\circ C = 373.15\ K$), R_v is the gas constant for water vapor, at $0\ ^\circ C$, R_v is approximately $461.5\ J/(kg \cdot K)$, L_v is the latent heat of vaporization (J/kg) at $0\ ^\circ C$ ($L_v \approx 2.5 \times 10^6 J/kg$), and P_∞ is the partial pressure of water vapor in the surrounding air (for mild climate conditions, $20\ ^\circ C$ and $50\ \%$ relative humidity, $P_\infty \approx 1170 Pa$.

The exponents n and m vary based on the flow regime: for laminar flow, $n \approx m \approx 0.5$; for turbulent flow, $n \approx m \approx 0.8$. The Reynolds number (Re) should be included to classify fluid flow regimes in this experiment(Smits, McKeon, & Marusic, 2011).

$$Re = \frac{\rho v D}{\mu}$$

Using this equation, the flow can be classified as follows:

- **Laminar flow:** $Re > 2300$
- **Transitional flow:** $2300 \leq Re < 4000$
- **Turbulent flow:** $Re \geq 4000$

Since $Re = 8459.945 > 4000$, the flow in this experiment should be classified as turbulent flow, making $n \approx m \approx 0.8$.

Substituting the above equations into the total heat loss integral, we can calculate the final temperature of the water:

$$\frac{dT}{dt} = -\frac{1}{m \cdot c}\left[h_0 \cdot v^n \cdot A \cdot (T_i - T_\infty) \cdot e^{-\alpha t} + \right.$$
$$\left. h_{m0} \cdot v^m \cdot A \cdot \left(P_0 \cdot \exp\left(\frac{L_v}{R_v} \cdot \left(\frac{1}{T_0} - \frac{1}{T_\infty + (T_i - T_\infty) \cdot e^{-\alpha t}}\right)\right) - P_\infty\right) \cdot L_v\right]$$

T_f is obviously subject to exponential decay. Thus, the relationship between T_f and v can be simplified as the following empirical equation:

$$T(v) = T_e + (T_i - T_e)\exp(-\alpha(v^n + v_0)t)$$

For further regression analysis, the equation can be written as:

$$T(v) = a + b\exp(-kv^n)$$

Let $w = \exp(-kv^n)$, the linearized equation can be written as:

$$T(v) = a + bw$$

1.5 Variables

Independent Variable: Airflow speed v (ms^{-1}), controlled by using different fans with multiple speed, measured with an anemometer

Dependent Variable: Final temperature of the water after five minutes T ($°C$), measured by a thermometer

Controlled Variables:

Variable	Why it needs to be controlled	How is it controlled
Initial Temperature (T_0)	Ensures that the cooling rate observed is due to the airflow speed, not differences in starting temperatures.	Water heated to 75 $°C$ before each trial. A precise thermometer was used to verify the starting temperature.
Room Temperature (T_∞)	Affects heat transfer from the water to the environment. Keeping it constant ensures variations in cooling are due to airflow speed and not fluctuations in ambient temperature.	The experiment was conducted in an air-conditioned room maintained at 24.3 $°C$. After each trial, the waiting period allowed for the temperature to return to 24.3 $°C$ before proceeding with the next trial.
Time of Cooling	Consistent cooling time allows for a fair comparison of the airflow speed's effect on the final temperature.	Set to be five minutes for all trials.

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Table 1 continued from previous page

Variable	**Why it needs to be controlled**	**How is it controlled**
The volume of Water	The volume influences the thermal mass, affecting how quickly it cools. Different volumes would result in different rates of temperature change.	200 ml water was used in each trial.
Type of Container	The material and shape of the container can affect the heat transfer rate. Different materials have different thermal conductivities, and the surface area exposed to air can vary with shape.	A ceramic bowl was used as the container of water. Ceramic has a relatively low specific heat capacity compared to water, minimizing its influence on the water's cooling rate. This allows the experiment to focus more accurately on the water's temperature change.
Airflow Pattern	The way air flows around the container can affect the cooling rate. Turbulent vs. laminar flow can lead to different heat transfer rates.	The wind source generates turbulent flows throughout the experiment. Minimum wind speed is 2.5 m s^{-1}. Reynolds number (Re) is kept well above 4000 (Smits et al., 2011), ensuring turbulent flow.

1.6 Apparatus and Methods

Table 2: Experimental Apparatus and Method

Apparatus	**Method**
• One Foam Box (1) • One Anemometer (2) • Diffuser (handmade with preservative film) (3) • Airflow Generator (two hairdryers and two fans with different modes to generate different wind speeds) (4) • One Counting Cup (5) • One thermometer (6) (For the actual experiment setup, see Figure 1)	1. Assemble the experimental setup as illustrated in Figure 1. Position the diffuser to ensure even airflow distribution. 2. Heat 200 mL of water to a starting temperature of 75°C. 3. Measure and record the room temperature. Confirm that the foam box is at room temperature to avoid external thermal influences. (This was done by leaving the experiment setup standing for 20 minutes after each trial).

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Table 2 – continued from previous page

Apparatus	Method
	Continued on next page 4. Adjust the airflow generator to the desired wind speed. Use the anemometer to measure the wind speed. 5. Place the heated water in the container within the foam box. Start the timer and allow the water to cool for exactly 5 minutes. 6. After 5 minutes, measure the final temperature of the water. 7. Repeat steps 2-6 for 21 times.

The numbers in parentheses (1)-(6) next to each apparatus item correspond to the numbered components in Figure 2, which provides a detailed illustration of the experimental apparatus.

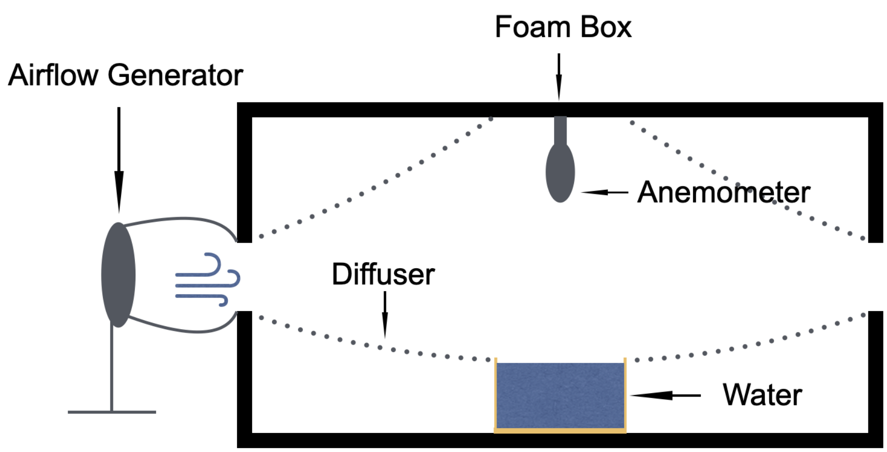

Figure 1: Experiment Setup

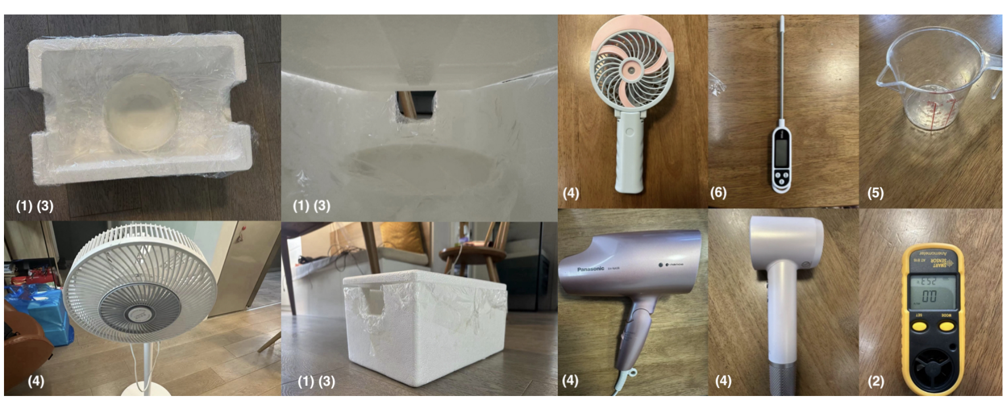

Figure 2: Experiment Setup

1.7 Ethical, Safety, and Environmental Concerns

Safety Concerns

- Handling hot water (up to 75°C) could pose a burn risk if spilled. Proper care should be taken when heating and transferring the water.
- Use of electrical equipment like fans near water could pose an electrical hazard if not properly managed.

Environmental concerns

- Energy consumption from repeatedly heating water and running fans/hairdryers. There may be some environmental impact from electricity usage.
- Water usage, though at $200mL$ per trial, should be considered.

Ethical concerns

- Since this experiment does not involve human or animal subjects, it has no major ethical concerns.

2 Analysis

2.1 Raw Data and Qualitative Observation

Below is the table for raw data without any calculation. Each set of data is repeated three times to reduce random error.

Airflow Speed (ms^{-1})	**Temperature (°C)**		
	Trial 1	**Trial 2**	**Trial 3**
0.00	48.3	48.8	47.9
2.50	43.9	44.0	43.7
3.60	42.5	42.1	41.5
7.10	40.0	40.5	39.5
9.40	38.7	38.4	38.5
17.6	33.5	33.9	33.6
19.3	32.1	31.6	32.4
21.4	29.3	29.6	28.9

Table 3: Raw Data

At first glance. there is a clear negative relationship between the airflow speed and the temperature after 300 seconds. Namely, as the airflow speed v increases, the temperature T decreases. This trend is consistent across all three trials, with minor variations. Initially, from 0 to about 7.1 m/s, the temperature drops steeply from around 48 $°C$ to $40°C$. Beyond this point, as airflow speed continues to rise, the temperature decline becomes more gradual, with temperatures ranging from approximately $33°C$ at 17.6 m/s to about $29°C$ at 21.4 m/s.

2.2 Non-linearized Regression

A sample calculation of trial 2 with an airflow speed of 9.40 $ms^{-1}$is shown below:

The mean of T for v of 9.40 ms^{-1} is

$$\bar{T} = \frac{\sum x_i}{n} = \frac{38.7 + 38.4 + 38.5}{3} = 38.3333 \approx 38.5°C$$

The value of uncertainty can be calculated as

$$\text{Uncertainty} = \frac{T_{max} - T_{min}}{2} = \frac{38.7 - 38.4}{2} = 0.150°C$$

$$T = 38.5 \pm 0.150$$

Applying this to all the data, the processed data is shown in the table below:

Table 4: Processed Data from Airflow Speed and Temperature

Airflow Speed (m s^{-1})	Temperature (°C)			Mean $\bar{x} = \frac{\sum x_i}{n}$	Uncertainty $\frac{T_{max}-T_{min}}{2}$
	Trial 1	Trial 2	Trial 3		
0 ± 0.0500	48.3	48.8	47.9	48.3	± 0.450
2.50 ± 0.0500	43.9	44.0	43.7	43.9	± 0.150
3.60 ± 0.0500	42.5	42.1	41.5	42.0	± 0.500
7.10 ± 0.0500	40.0	40.5	39.5	40.0	± 0.500

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Table 4 – continued from previous page

Airflow Speed ($m\ s^{-1}$)	Temperature (°C)			Mean $\bar{x} = \frac{\sum x_i}{n}$	Uncertainty $\frac{T_{max}-T_{min}}{2}$
	Trial 1	Trial 2	Trial 3		
9.40 ± 0.0500	38.7	38.4	38.5	38.5	$\pm$ 0.150
17.6 ± 0.0500	33.5	33.9	33.6	33.7	$\pm$ 0.200
19.3 ± 0.0500	32.1	31.6	32.4	32.0	$\pm$ 0.400
21.4 ± 0.0500	29.3	29.6	28.9	29.3	$\pm$ 0.350

The uncertainty for the airflow speed is considered to be $0.0500ms^{-1}$, as the minimum scale of the anemometer is $0.1ms^{-1}$.

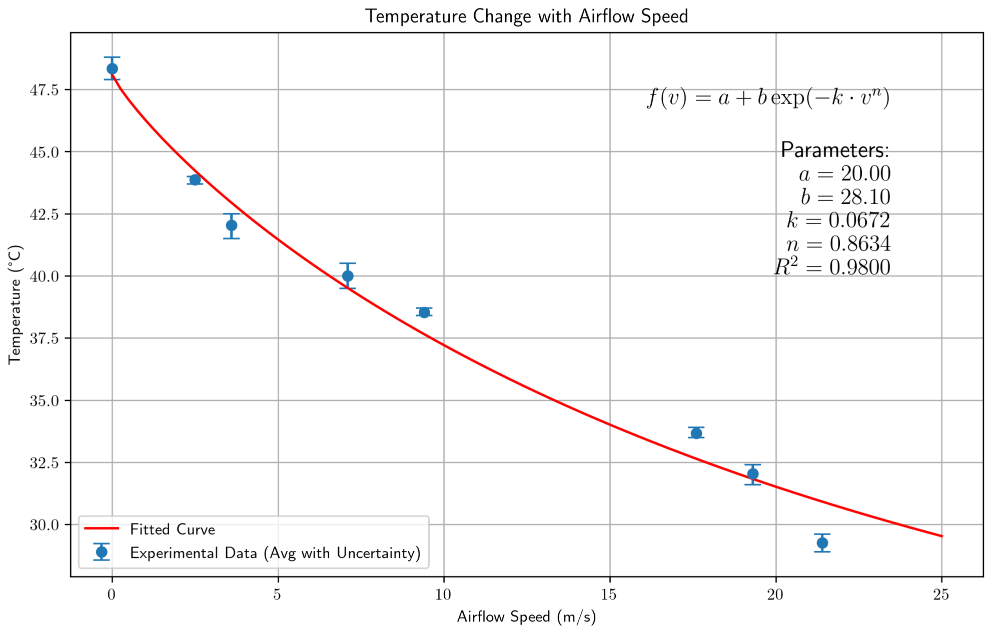

Figure 3: Temperature Change with Airflow Speed

This graph illustrates the relationship between T_f and v, along with the fitted data results. The fit quality is excellent, as evidenced by the high R^2 value of 0.9800. The curve exhibits characteristics of exponential decay, which aligns with the previous deduction.

Notably, the graph lacks a theoretical curve, which is not uncommon in fluid dynamics research. Many fluid dynamic phenomena are yet impossible to predict precisely from theory, making empirical formulas often more practical. This experiment successfully reveals the quantitative relationship between temperature and airflow speed.

Despite this, the fitted parameters demonstrate a strong correlation with physical expectations and theoretical predictions. The airflow speed exponent n of 0.8634 indicates turbulent flow conditions, consistent with the previously calculated result. The equilibrium temperature T_e at 20.0 $°C$ closely approximates the ambient temperature of 24.3 $°C$. The equivalent wind speed v_0 of 10.0 m/s for natural convection and evaporation effects is reasonable. The cooling rate coefficient $k = 0.0672$ is a regression parameter that cannot be measured in reality.

2.3 Linearized Regression

A sample calculation of airflow speed of $19.3 \pm 0.0500 ms^{-1}$is shown below:

$$w = e^{-kv^n}$$

Based on the previous regression, $k = 0.0672$ and $n = 0.8634$. Hence the value of w for $v = 19.3$ is

$$w = e^{-kv^n} = e^{-0.0672 \times 19.3^0.8634} \approx 0.421$$

The max-min method will be used to calculate the propagation of error for w.

$$w_{max} = e^{-kv_{min}^n} = e^{-0.0672 \times 19.25^0.8634} = 0.4216128 \approx 0.422$$

$$w_{min} = e^{-kv_{max}^n} = e^{-0.0672 \times 19.35^0.8634} = 0.4199833 \approx 0.420$$

$$\Delta w = \frac{w_{max} - w_{min}}{2} = 0.00100$$

Applying this to all the data, the processed data is shown below:

Table 5: Relationship between airflow velocity and the dimensionless parameter w with associated uncertainties.

Airflow Velocity $v \pm \Delta v$ (m/s)	w $w = e^{-kv^n}$	$w_{\mathbf{max}}$ $w_{\min} = e^{-k(v+\Delta v)^n}$	$w_{\mathbf{min}}$ $w_{\max} = e^{-k(v-\Delta v)^n}$	**Uncertainty in** w $\frac{w_{\max}-w_{\min}}{2}$
0.00 ± 0.0500	1.000	0.995	1.000*	0.003
2.50 ± 0.0500	0.862	0.860	0.864	0.00200
3.60 ± 0.0500	0.816	0.814	0.818	0.00200
7.10 ± 0.0500	0.694	0.693	0.696	0.00200
9.40 ± 0.0500	0.628	0.627	0.629	0.00100
17.60 ± 0.0500	0.450	0.449	0.450	0.00100
19.30 ± 0.0500	0.421	0.420	0.422	0.00100
21.40 ± 0.0500	0.388	0.387	0.389	0.00100

Constants: $k = 0.0672$, $n = 0.8634$
*For $v = 0$, $w_{\min}$ is set to 1 due to the undefined nature of $(-0.0500)^{0.8634}$ in the complex domain when calculating $e^{-k(0-\Delta v)^n}$.

The graph demonstrates a robust linear relationship between these variables, suggesting airflow speed has a significant impact on the final temperature in this experiment. Notably, when $w = 0.629$, the data point appears to be an outlier. However, upon examination of the overall dataset, there is no significant systematic deviation from the trend line, indicating that this anomaly is most likely due to random error rather than systematic error.

The equation of the best-fit line is:

$$y = 28.19 + 19.93w$$

Despite this, the error bars appear unusually small, leading to sub-optimal maximum and minimum slope fits. Ideally, these lines should fall within the error bars for all data points, which is not the case here.

The max-min method is used for the calculation of the uncertainties of slope and y-intercept.

The absolute uncertainty of slope Δb:

$$\Delta b = \frac{b_{\max} - b_{\min}}{2} = \frac{31.01 - 25.37}{2} = 2.82((m/s)^{0.8634})$$

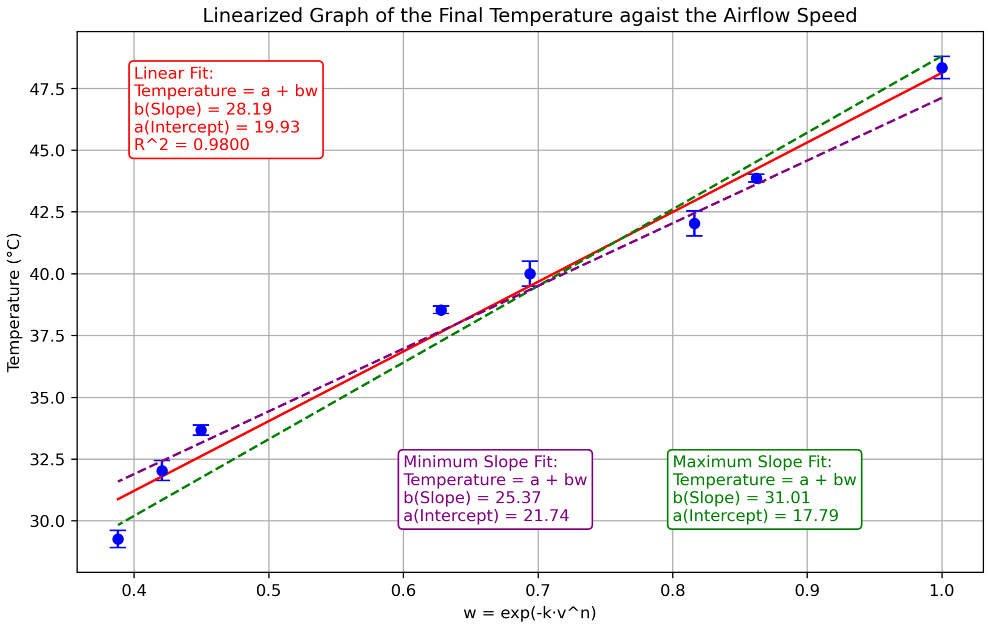

Figure 4: Linearized Graph of the Final Temperature against the Airflow Speed

The percentage uncertainty of slope $\%\Delta b$:

$$\%\Delta b = \frac{2.82}{28.19} \approx 10.0\%$$

The absolute uncertainty of y-intercept Δa:

$$\Delta a = \frac{a_{\max} - a_{\min}}{2} = \frac{21.74 - 17.79}{2} = 1.975 \approx 1.98(K)$$

The percentage uncertainty of y-intercept $\%\Delta a$:

$$\%\Delta a = \frac{1.98}{19.93} \approx 9.93\%$$

2.4 Conclusion

This analysis validates the previously derived exponential decay relationship between airflow speed and the final temperature of the water sample.

However, the unusually small error bars graph suggests that other error propagation methods should be used. While there is an outlier in the data points, it's important to note that empirical formulas in fluid dynamics rarely produce perfectly accurate predictions, and this level of deviation is acceptable without compromising the overall reliability.

Notably, the regressed parameters closely align with real-world physical conditions. For instance, the equilibrium temperature $T_e(20.00°C)$ approximates the ambient temperature $(24.3°C)$, and the airflow speed exponent n (0.8634) indicates turbulent flow conditions, consistent with previous calculations.

3 Evaluation

3.1 Strengths

Despite outliers and random errors, this investigation has many strengths:

1. The experimental setup is well-designed. The apparatus effectively isolates the effect of airflow speed on cooling. The foam box minimizes external thermal influences, while the custom-made diffuser ensures even airflow distribution around the water container.

2. Variables are well-controlled. Maintaining consistent initial water temperature, volume, cooling time, and ambient room temperature across trials helps isolate the impact of airflow speed.

3. This experiment has a strong and comprehensive theoretical foundation. A sophisticated heat balance equation is developed that incorporates both convective heat transfer and evaporative cooling.

4. Three trials for each airflow speed are conducted. This significantly reduces random error and increases result reliability.

3.2 Weakness

Error Source	**Effects Significance**	**Evidence**	**Improvements**
Assumption of uniform airflow	Intermediate	The setup uses a diffuser, which helps distribute airflow, but perfect uniformity is unlikely. This could explain some of the data variability observed.	Use multiple anemometers placed at different points around the container to measure and ensure airflow uniformity. Average these readings for accuracy.
Room temperature fluctuations	Intermediate	The room temperature is maintained at 24.3 $°C$. but the precision is not specified. Small fluctuations could affect cooling rates.	Continuously monitor and record room temperature throughout each trial. Normalize results if significant fluctuations occur.
Water evaporation	Significant	The evaporative cooling is included in the framework, but the experimental procedure doesn't measure or control for water loss. This could lead to overestimations.	Measure water volume or mass before and after each trial to quantify evaporation. Use this data to adjust cooling rate calculations. Consider using a lid.
Temperature stratification in water	Significant	The report doesn't mention stirring or circulation of water during cooling. Stratification could lead to misleading temperature readings.	Gently stir the water before each measurement or use multiple probes at different depths and average the readings.

Error Source	**Effects Significance**	**Evidence**	**Improvements**
Anemometer placement	Significant	Anemometer is held by myself for each trial as it cannot be tightly fixed. Improper positioning could result in inaccurate readings.	Standardize anemometer placement, ideally measuring at multiple points and at the same height as the water surface. Average these readings for accuracy.
Water purity variations	Insignificant	Water source and purity are not controlled. While likely minor, variations could introduce inconsistencies.	Use distilled or deionized water for all trials.

Table 6: Summary of potential error sources, their significance, evidence, and suggested improvements for the experimental setup.

3.3 Extension

An intriguing extension of this experiment could explore cooling dynamics across various conditions. For example, by extending observation time to record complete cooling curves until ambient temperature is reached, deeper insights into the stages of the cooling process and changes in cooling rates over time could be gained. Also, the cooling method can be changed to compare with forced convection conditions. Simultaneously, the impact of container materials (e.g., metals, plastics) on heat conduction rates can be examined. Generally speaking, most of the controlled variables in this experiment can be the focus of further research.

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